Multiple Strategies to Analyze Monty Hall Problem

There is a tendency to approach a new problem from a single perspective, often an intuitive one. The first step is to recognize this tendency and take a step back. An unexamined assumption that is incorrect can take you down the wrong path. It not only consumes time and resources, but it is difficult to go all the way back to the beginning.

The Monty Hall Problem is a good example. The intuitive response is incorrect. A fact missed by the majority of people, including trained mathematicians.

Once the assumption is recognized, it must be critically analyzed. Although individuals have their favorite strategy, the more approaches that you can consider for a particular problem, the greater the likelihood for success.

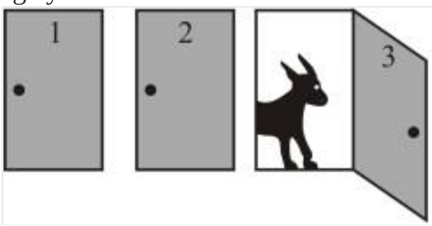
4 Approaches to the Monty Hall Problem

- Diagrams
  - Used previously as Concept Maps
- Probability Tree
  - Similar to Reason's Risk Management Model
- Bayes Theorem
  - Used in Cancer Problem
- Numerical Simulation
  - NASA Rocket Simulator
  - JMU Truss Analysis

The Monty Hall problem in probability (Diagrams)

http://www.cuetutors.com/the-monty-hall-problem-in-probability/

Suppose you are in a game show, and you're given the choice of three doors. Behind one door is a new car; behind the others, goats. The car and the goats were placed randomly behind the doors before the show. The rules of the game show are as follows: After you've chosen a door, the door remains closed for the time being. The game show host, Monty Hall, who knows what is behind the doors, now has to open one of the two remaining doors, and the door he opens must have a goat behind it. If both remaining doors have goats behind them, he chooses one randomly. After Monty Hall opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door. Imagine that you choose Door 1 and the host opens Door 3, which has a goat. He then asks you "Do you want to switch to Door 2?" Is it to your advantage to change your choice?



In search of a new car, the player picks door 1. The game host then opens door 3 to reveal a goat and offers to let the player pick door 2 instead of door 1.

Before going through the solution, give the problem a great deal of thought. Will switching change the probability of winning? Or will it not matter because it is equally likely that the car may be behind Door 1 or Door 2 ?
O.K., now the solution. We are assuming that the player initially picked Door 1. (However, the player may initially choose any of the three doors). We'll now calculate the probability of winning by switching, by listing out all cases explicitly.

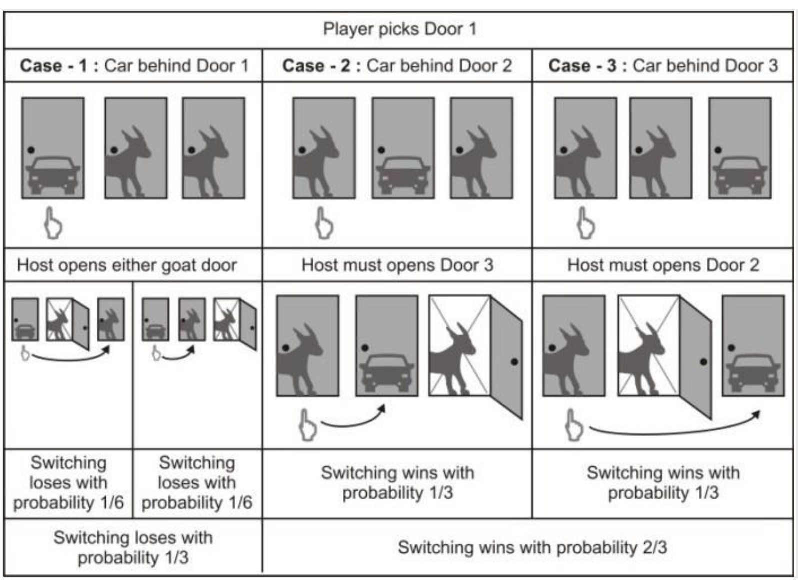
Case 1: The car is behind Door 1. In this case, the game host Monty Hall must open one of the two remaining doors randomly

Case 2: The car is behind Door 2. Then the host must open Door 3

Case 3: The car is behind Door 3. Then the host must open Door 2

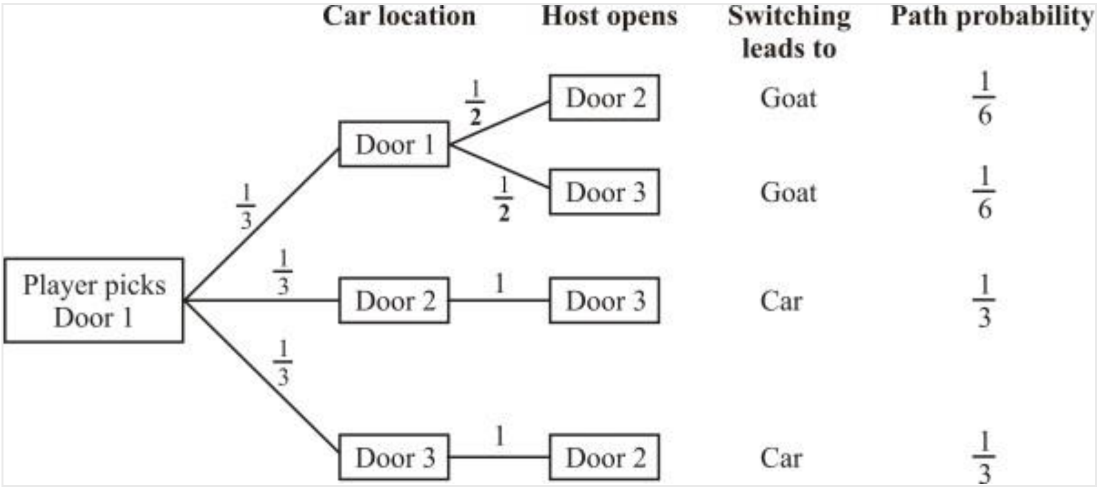
If the player chooses to switch, he'll win only if the car is behind either of the two unchosen doors rather than the one that was originally picked.

The following diagram visually depicts the various cases and what happens with switching.



Probability Tree

Let us explain how we arrived at the various probabilities using a probability tree:



Thus, switching leads to winning with probability 2/3 , that is, the player should switch for a higher probability of winning! This result may seem very counter-intuitive to you. After all, you may say, the remaining two doors must each have a probability 1/2 of containing the car. However, this intuition is wrong.

The Monty Hall Problem: Bayes Theorem

http://www.cs.dartmouth.edu/~afra/goodies/monty.pdf
modified from Afra Zomorodian, January 20, 1998

Introduction

This is a short report about the infamous "Monty Hall Problem." The report contains two solutions to the problem: an analytic and a numerical one. The analytic solution will use probability theory and corresponds to a mathematician's point of view in solving problems. The numerical solution simulates the problem on a large scale to arrive at the solution and therefore corresponds to a computer scientist's point of view.

The Monty Hall Problem

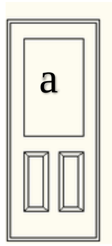
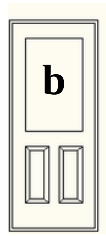
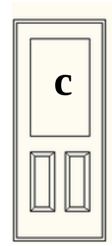
The Monty Hall Problem's origin is from the TV show, "Let's Make A Deal" hosted by Monty Hall. The statement of the problem is as follows [LR94]:

"You are a contestant in a game show in which a prize is hidden behind one of three doors. You will win the prize if you select the correct door. After you have picked one door but before the door is lifted, the emcee lifts one of the other doors, revealing an empty stage, and asks if you would like to switch from your current selection to the remaining door. How will your chances change if you switch?"

The question was originally proposed by a reader of "Ask Marilyn", a column by Marilyn vos Savant in Parade Magazine in 1990 and her solution caused an uproar among Mathematicians, as the answer to the problem is unintuitive: while most people would respond that switching should not matter, **the contestant's chances for winning in fact double if she switches curtains.** Part of the controversy, however, was caused by the lack of agreement on the statement of the problem itself. We will use the above version. For accounts of the controversy as well as solutions and interactive applets, see [Don98].

Proof by Bayes Theorem

http://www.cs.dartmouth.edu/~afra/goodies/monty.pdf (modified from Afra Zomorodian)

Doors a,b,c Event that each door is opened a, b, c O_B is the probability that the host opens door B			
Prior Probability that car is behind door	Pr {A} = 1/3	Pr{B} = 1/3	Pr{C} = 1/3
Contestant Picks A			
Probability of host opening door b for the prize being behind each door	$Pr\{O_B \mid A\} = 1/2$, if the prize is behind *a*, host can open either *b* or *c*	$Pr\{O_B \mid B\} = 0$, if the prize is behind *b*, host cannot open door *b*	$Pr\{O_B \mid C\} = 1$, if the prize is behind *c*, host can only open curtain *b*

In order to determine whether to switch your selection from Door a to Door b after seeing the new information is revealed when t there is no car behind door b.

The reason for switching can be restated as:

Is $\Pr\{A \mid O_B\}$ (stay with door a) $<$ $\Pr\{C \mid O_B\}$? (switch to door c)

$\Pr(O_B)$ = sum of the probabilities of the doors a + b + c (the prize can be behind one curtain only)

$$\begin{aligned} Pr\{O_B\} &= Pr\{A\}Pr\{O_B \mid A\} + Pr\{B\}Pr\{O_B \mid B\} + Pr\{C\}Pr\{O_B \mid C\} \\ &= (1/3)\cdot(1/2) \quad + \quad (1/3)\cdot 0 \quad + \quad (1/3)\cdot 1 \\ &= 1/2. \end{aligned}$$

By Bayes Theorem:

Probability that car is behind door a if host opens door b

$$\Pr\{A/O_B\} = \frac{\Pr\{A\}\, Pr\{O_B \mid A\}}{Pr\{O_B\}} = \frac{(1/3)\ (1/2)}{1/2} = 1/3$$ (note that this is unchanged from Pr{A}

Probability that car is behind door c if host opens door b

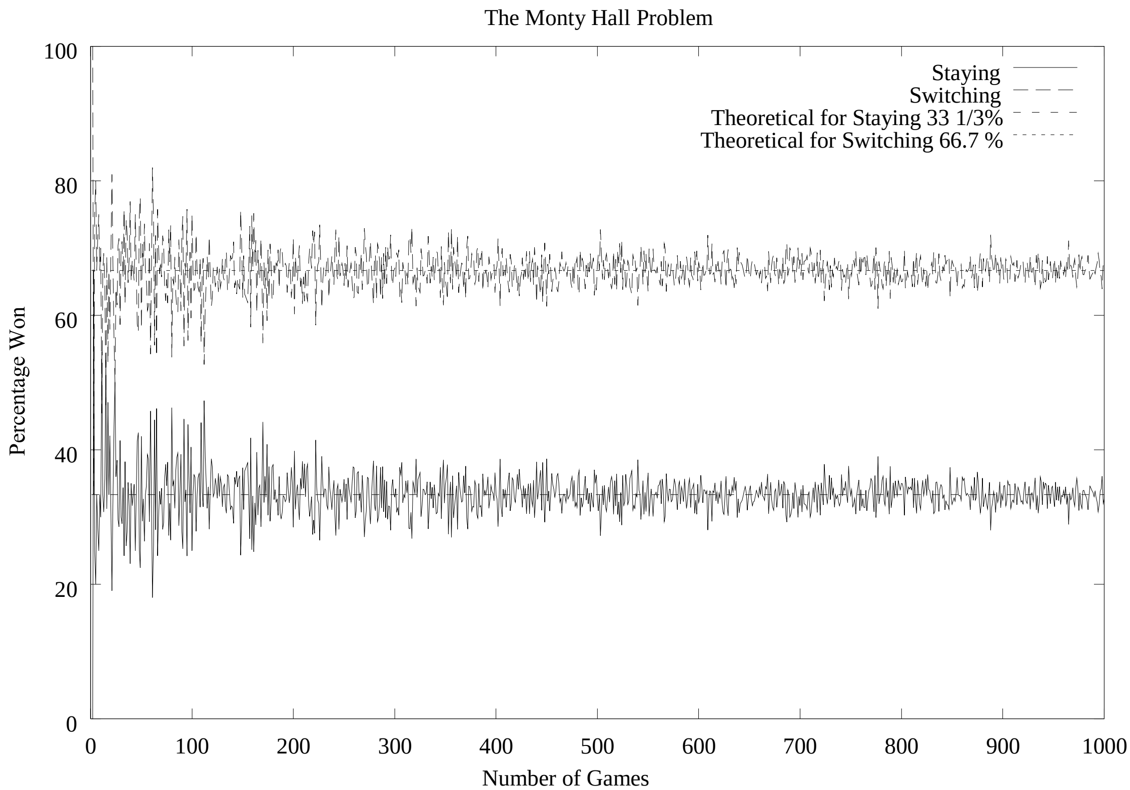
$$\Pr\{C/O_B\} = \frac{\Pr\{C\}\, Pr\{O_B \mid C\}}{Pr\{O_B\}} = \frac{(1/3)\ (1)}{1/2} = 2/3$$ (note that this is twice the original Pr{C}

Compare the probabilities of the price being behind door a compared to door b

$\Pr\{A/O_B]$ < $\Pr\{C/O_B\}$ by a factor of 2

In other words, if you switch, your chances of winning are **doubled**.

Solution by Simulation



A program for simulating the generalized Monty Hall Problem was implemented in ANSI-C and is included. Using the program, data was generated for the two following graphs.

The graph above shows the convergence of the probabilities to the theoretical values for 3 curtains (each set of game was independently run.) The graph plots the percentage of winning if the contestant always chooses to stay with the original door versus if the contestant chooses to switch. The graph clearly reaffirms the theoretical result.

References

[Don98] Dennis Donovan. The WWW Tackles The Monty Hall Problem, 1998. http://math.rice.edu/~ddonovan/montyurl.html.

[LR94] Thomas H. Cormen, Charles E. Leiserson, and Ronald L. Rivest. *Introduction to Algorithms*. The MIT Press, Cambridge, Massachusetts, 1994